TRINITY COLLEGE

Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	10	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(7 marks)**

Sam is given the task of archiving 948 newspapers. On the first day he archived 5 newspapers, on the second day he archived 8 newspapers and on each subsequent day he archived 3 more newspapers than he did the day before.

(a) Determine

(i) how many newspapers Sam archived on the 5th day. (1 mark)

Solution
$T_5 = 17\ newspapers$
Specific behaviours
✓ correct amount

(ii) how many papers Sam had archived altogether after the 5th day. (1 mark)

Solution
$S_5 = 55$
Specific behaviours
✓ correct amount

(b) On which day did Sam finish the task of archiving the newspapers and how many did he archive on that day? (2 mark)

Solution
$T_{24} = 74, \quad S_{24} = 948$ Sam finished on the 24th day, archiving 74 newspapers that day.
Specific behaviours
✓ correct day ✓ correct amount

After archiving the newspapers, Sam is given the task of refilling 50 toner cartridges. On the first day he refills 4 cartridges, leaving 46 remaining to be refilled, and continues to refill 4 cartridges on every subsequent day.

(c) (i) Deduce a simplified rule for the number of cartridges remaining to be filled at the end of the nth day. (2 marks)

Solution
$C_n = 46 - 4(n-1)$ $C_n = 50 - 4n$
Specific behaviours
✓ correct *a* and *d* value ✓ simplifies rule

(ii) Hence, or otherwise, determine the number of cartridges that remain to be filled at the end of the 6th day. (1 mark)

Solution
$C_n = 50 - 4(6) = 26$ Sam needs to refill 26 cartridges
Specific behaviours
✓ subs *n* = 6 into rule to determine value

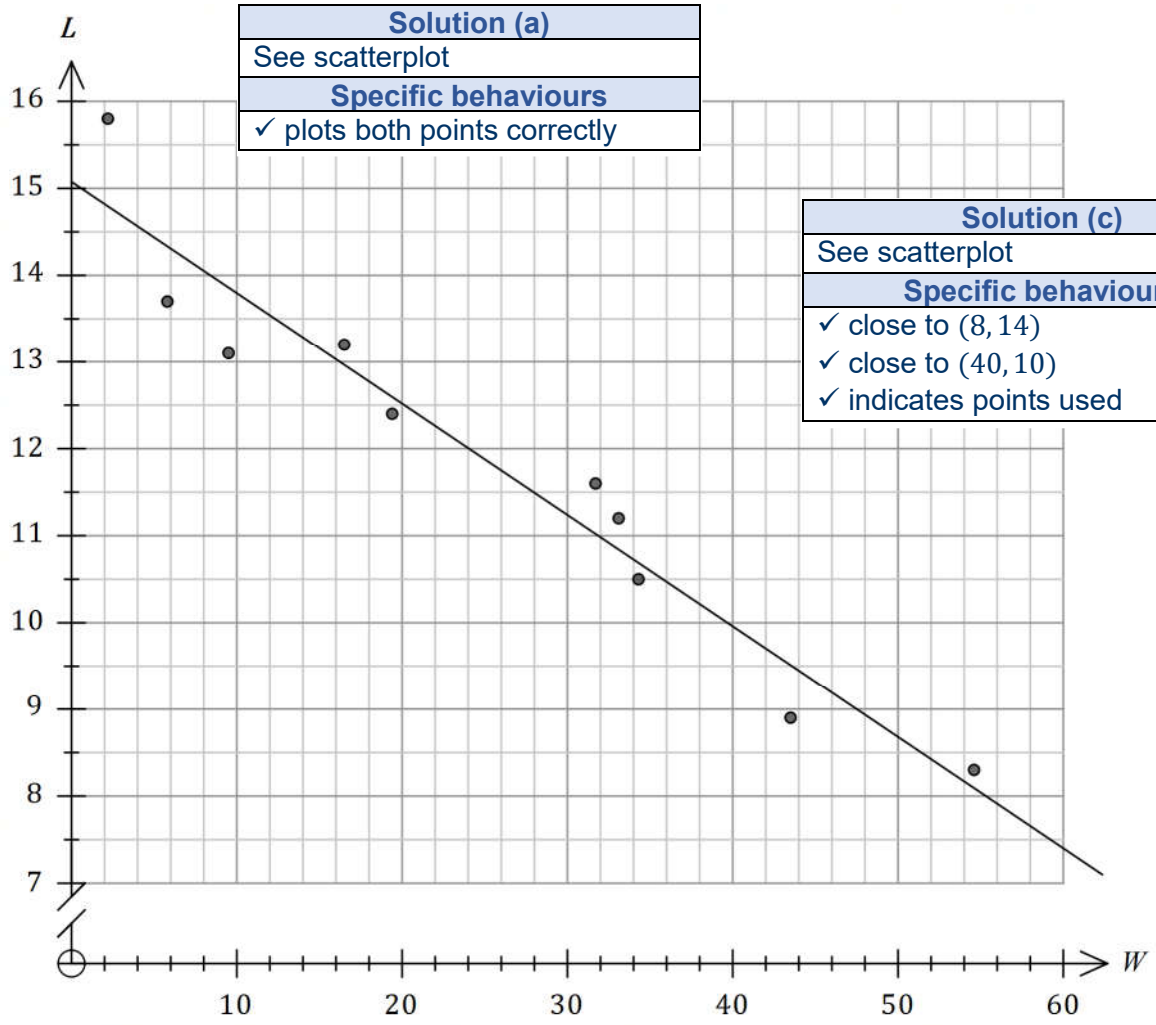
See next page

Question 10 **(10 marks)**

The table below shows the average lifespan L years and the average adult weight W kg of male dogs for a variety of breeds.

Breed	Weight W (kg)	Lifespan L (years)
English Setter	33.1	11.2
Jack Russell	5.8	13.7
Saint Bernard	43.5	8.9
Chihuahua	2.2	15.8
Beagle	9.5	13.1
Bullmastiff	54.6	8.3
Golden Retriever	31.7	11.6
Border Collie	16.5	13.2
German Shepherd	**34.3**	**10.5**
Spaniel	**19.4**	**12.4**

(a) Complete the scatterplot below by plotting the data for German Shepherd's and Spaniel's. Clearly indicate these points on your graph. (1 mark)



Solution (a)
See scatterplot
Specific behaviours
✓ plots both points correctly

Solution (c)
See scatterplot
Specific behaviours
✓ close to $(8, 14)$
✓ close to $(40, 10)$
✓ indicates points used

See next page

(b) Determine

(i) the correlation coefficient between W and L. (1 mark)

Solution
$r = -0.967$
Specific behaviours
✓ coefficient to at least 2 dp

(ii) the equation of the least-squares line that can be used to predict L from W. (2 marks)

Solution
$L = -0.128W + 15.08$
Specific behaviours
✓ coefficients ✓ uses correct variables

(c) Add the least-squares line to the scatterplot, clearly indicating the points used. (3 marks)

(d) A breed of dog has an average male weight of 52.5 kg.

(i) Predict the average lifespan of males of this breed. (1 mark)

Solution
$L(52.5) = 8.4$ years
Specific behaviours
✓ lifespan

(ii) Briefly discuss two factors that support the validity of your prediction. (2 marks)

Solution
Correlation: The strength of the relationship between the two variables is very strong, with $r = -0.967$. Interpolation: The prediction involves interpolation, since the weight of 52.5 kg lies between 2.2 and 54.6 kg.
Specific behaviours
✓ indicates strong correlation ✓ indicates interpolation

See next page

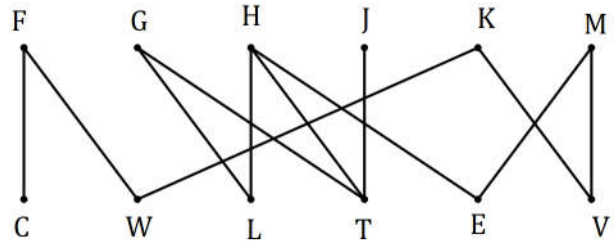
Question 11 **(7 marks)**

Six students were asked to create a short presentation to explain the meaning of some graph theory terms. The following table shows which terms each student offered to present.

Student	Terms
Fred	Circuit, Walk
Grace	Loop, Trail
Hazel	Loop, Trail, Edge
Joe	Trail
Kavan	Walk, Vertex
Mia	Edge, Vertex

(a) Draw a bipartite graph to represent this information. (3 marks)

Solution



Specific behaviours

✓ two distinct sets of vertices
✓ at least 9 edges correctly shown
✓ all 12 edges correctly shown

(b) Determine how many more edges must be added to the bipartite graph in (a) so that it would be a complete bipartite graph. (2 marks)

Solution

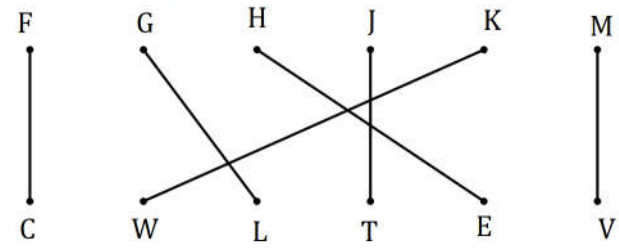
Complete bipartite will have $6 \times 6 = 36$ edges.
Already have 12 edges, so need another 24 edges.

Specific behaviours

✓ indicates total edges required
✓ correct number to add

(c) Draw another bipartite graph to show how it is possible to assign each student to present just one term, so that all six terms are explained. (2 marks)

Solution



Specific behaviours

✓ all vertices of order one
✓ correct pairings

See next page

Question 12 **(8 marks)**

A random sample of 42 drivers was taken at a test centre. Each pair of letters shown below represents one driver. The first letter shows the driving test outcome (Pass, P or Fail, F) for the person and the second letter shows if they were taking the test for the first time (Yes, Y or N, No).

FN PN FN FY PY PN PY FY FN PY PN PY PY PN FY PY PN PN FN PN PN

FN FN PY FY PY FN FN FN FY PY FN FN PY FN PN PY PY **PN FY FY FN**

(a) Two categorical variables have been recorded for each driver. Name one of the variables and explain why it is categorical. (2 marks)

Solution
One variable is driving test outcome and it is categorical because the outcomes are categories - pass or fail.
Specific behaviours
✓ names a variable ✓ explains categorical

(b) The two-way frequency table below summarises the data. Complete the table by adding the last four results (bolded above) to the table. (2 marks)

	Yes	No
Pass	**12**	**10**
Fail	**7**	**13**

Solution
See table
Specific behaviours
✓ two correct entry ✓ all correct entries and states totals

(c) Convert the two-way frequency table to show column percentages. (2 marks)

%	Yes	No
Pass	**63**	**43**
Fail	**37**	**57**

Solution
See table
Specific behaviours
✓ one correct percentage ✓ all correct percentages

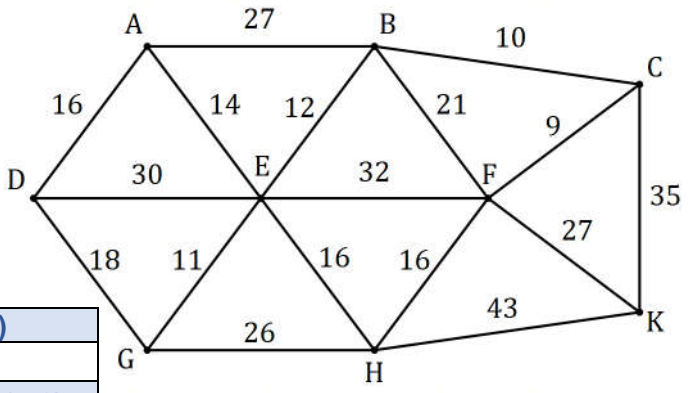
(d) Discuss whether this sample data suggests the presence of an association between passing the driving test and taking the test for the first time. (2 marks)

Solution
Yes, an association is present. The percentages in the Pass row are quite different, indicating that a higher percentage of drivers pass the test on their first attempt (63%) than if they are repeating (43%).
Specific behaviours
✓ indicates association present ✓ justifies by comparing different row percentages

See next page

Question 13 **(7 marks)**

In the graph below, the vertices represent towns and the weights on each edge represent the distance, in kilometres, between pairs of towns. A parcel delivery service is based at town D.



Solution (a)
See table
Specific behaviours
✓ 0.5 each, round down

(a) Complete the table below to show the shortest distance d km from town D to each of the other towns. (4 marks)

Town	A	B	C	E	F	G	H	K
d, km	**16**	**41**	**51**	**29**	**60**	**18**	**44**	**86**

(b) State the route that gives the minimum distance between towns D and K. (1 mark)

Solution
D-G-E-B-C-K
Specific behaviours
✓ correct route

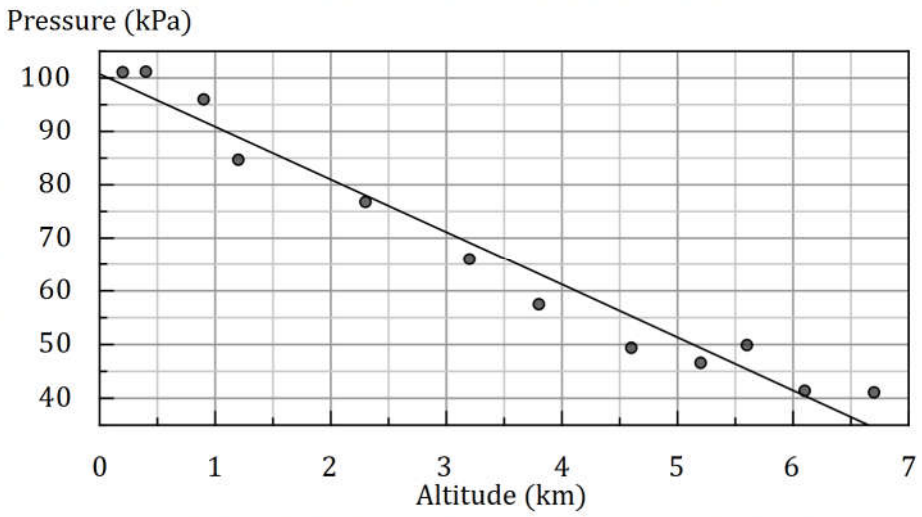
(c) One day the delivery service has two parcels to deliver, one at A and the other at K. Determine the shortest path from D to K that passes through A and state the length of this path. (2 marks)

Solution
D-A-E-B-C-K Distance is 87 km.
Specific behaviours
✓ correct route ✓ correct distance

See next page

Question 14 **(7 marks)**

The graph below shows pressure and altitude readings collected from a variety of sites in a country, together with the least-squares line for the linear association between the variables.



(a) The correlation coefficient r for the linear association is one of the values shown in the list below. Circle this value and justify your choice. (3 marks)

$$\{-0.98, \quad -0.82, \quad -0.35, \quad 0.03, \quad 0.35, \quad 0.82, \quad 0.98\}$$

Solution
Circles -0.98. r must be close to -1 to reflect the strong, negative association.
Specific behaviours
✓ circles correct value ✓ indicates negative direction ✓ indicates strong association

(b) Determine the coefficient of determination for the linear association and interpret its value. (2 marks)

Solution
$r^2 = (-0.98)^2 = 0.96$ 96% of the variation in the pressures at the sites can be explained by the variation in their altitudes.
Specific behaviours
✓ correct value (decimal or percentage) ✓ correct interpretation

(c) State, with reasons, whether the nature of the relationship between the variables is linear or non-linear. (2 marks)

Solution
Non-linear - 'points lie close to a curve' or 'a pattern would be evident in a residual plot for linear model' (+ve residuals, then -ve, then +ve again).
Specific behaviours
✓ states non-linear ✓ reason to support non-linear

See next page

Question 15 **(8 marks)**

A photocopier was purchased for $\$3\,250$. Its value depreciates at a rate of 4.5 cents per copy. The following recurrence relation can be used to determine V_n, the value of the photocopier in dollars after n copies have been made.

$$V_n = 3250 + b(n)$$

(a) Explain why the value of b is -0.045. (2 marks)

Solution
b is negative because the value is decreasing, it is 0.045 as that is 4.5 cents in dollars
Specific behaviours
✓ explains why b is negative ✓ explains why b is 0.045

(b) Determine V_{400}. (1 mark)

Solution
$V_{400} = 3250 - 0.045 \times 400$ $= \$3\,232$
Specific behaviours
✓ correct value

(c) Determine n when $V_n = 2647$. (2 marks)

Solution
$2647 = 3250 - 0.045n$ $n = 13\,400$
Specific behaviours
✓ correct equation ✓ correct value of n

(d) Calculate the value of the photocopier after $44\,000$ copies have been made. (1 mark)

Solution
$V_{44000} = 3250 - 0.045 \times 44000$ $= \$1\,270$
Specific behaviours
✓ correct value

(e) The photocopier will be replaced as soon as its value falls below $\$350$. Determine the number of copies the photocopier will make before it is replaced. (2 marks)

Solution
$350 = 3250 - 0.045n$ $n = 64444.\overline{4}$ Will be replaced after $64\,445$ copies.
Specific behaviours
✓ correct equation ✓ correct number of copies

See next page

Question 16 **(6 marks)**

The scores of a sample of students who sat two tests are shown in the table below.

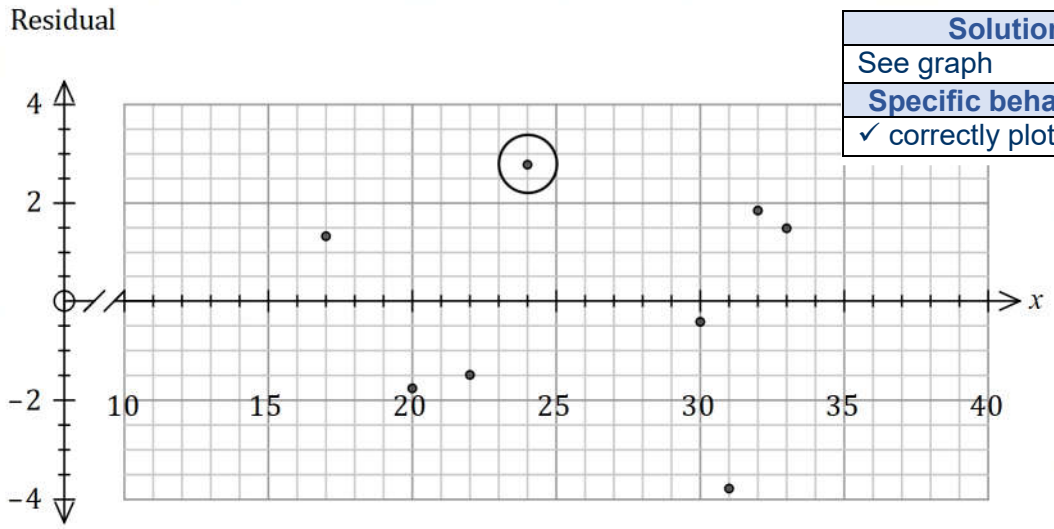
Student	1	2	3	4	5	6	7	8
Test A	33	22	30	24	17	20	32	31
Test B	14	18	14	21	24	19	15	10
Residual	1.49	-1.49	-0.42	p	1.33	-1.76	1.85	-3.78

Two students missed Test B and their teacher planned to predict their marks for this test using their scores from Test A and the linear relationship modelled by the least-squares line between the response (y) and explanatory (x) variables.
The equation is $y = -0.635x + 33.46$ and the correlation coefficient is -0.867. This equation was used to determine the residuals shown in the table above.

(a) Determine the value of p in the table above. (2 marks)

Solution
$y = -0.635(24) + 33.46 = 18.22$ $p = 21 - 18.22 = 2.78$
Specific behaviours
✓ correct value of y ✓ correct residual

(b) Complete the residual plot by adding your residual from (a) to the plot below. (1 mark)



Solution
See graph
Specific behaviours
✓ correctly plots point

(c) Using the residual plot and other relevant factors, comment on the teacher's plan. (3 marks)

Solution
The teachers plan is sound as (i) no pattern evident in residuals and so use of linear model is appropriate. (ii) the linear relationship is strong, with $r = -0.867$ However, the scores of the missing students in Test A are unknown. If they are not between 17 and 33 then the predictions for Test B will involve extrapolation and should be treated with caution.
Specific behaviours
✓ indicates no pattern in residuals is good ✓ indicates strong correlation is good ✓ indicates possible danger of extrapolation

See next page

Question 17 **(10 marks)**

Forrest John national park is home to a population of koalas whose numbers are increasing at a rate of 7.5% per year. In 2005 park rangers began a long-term study to monitor the changing number of koalas in the national park. They determined the initial population of koalas in 2005 to be 1600.

(a) State a recurrence relation for the koala population *n* years after 2005. (2 marks)

Solution
$T_{n+1} = 1.075T_n, \quad T_0 = 1600$
Specific behaviours
✓ correct recursive rule
✓ states T_0

(b) (i) Determine the koala population in 2010. (1 mark)

Solution
$T_5 = 2297.006921..$
There will be 2297 koalas in the park
Specific behaviours
✓ states T_5 value

(ii) Determine, to the nearest year, how long it will take for the koala population to approximately double. (2 marks)

Solution
$1600 \times 2 = 3200, \quad T_{10} = 3297.65$
It will take approximately 10 years for the population to double.
Specific behaviours
✓ states double the population is 3200
✓ determines the correct year population will double

(c) What assumption has been made in the calculations in parts (b)? (1 mark)

Solution
The calculations assume that the rate of growth will remain the same.
Specific behaviours
✓ any reasonable assumption

A bush fire at the end of 2010 in Forrest John national park reduces the amount of food available to the koalas. It is estimated that the population will begin to decrease at a rate of 17% per annum.

(d) Using your 2010 population from part (b)(ii) estimate the population of koalas in the park in 2018. (2 marks)

Solution
$P_{n+1} = 0.83P_n, \quad P_0 = 2297$
$P_8 = 517$ There will be 517 koalas in the park.
Specific behaviours
✓ uses value from (b)(i) as T_0
✓ determines correct population of koalas

Starting in 2019, to combat the decline of koalas, park rangers decided to introduce 255 koalas to the park each year.

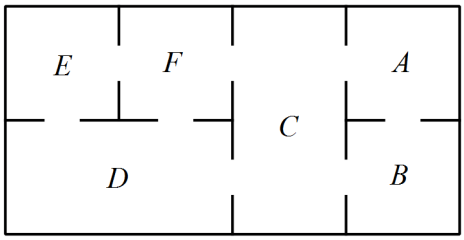
(e) What is the expected long-term population of koalas? (2 mark)

Solution
$P_{n+1} = 0.83P_n + 255, where\ P_{n+1} = P_n$
$x = 0.83x + 255 \ \therefore x = 1500$
The expected long-term population is 1500 koalas
Specific behaviours
✓ indicates $P_{n+1} = P_n$
✓ determines correct long-term population of koalas

See next page

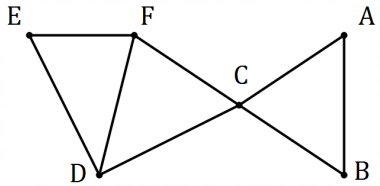
Question 18 **(8 marks)**

A warehouse has dividing walls that split its interior into six areas, as shown in the plan below. The gaps in the dividing walls are doorways that allow people to move from one area to another.



(a) Construct a graph to represent the warehouse areas and doorways, with each area being a vertex and each doorway an edge. (2 marks)

Solution



Specific behaviours

✓ connected graph, 6 vertices, labels
✓ correct graph

(b) An inspector started in one area and followed a route that went through all doorways exactly once before stopping in another area. State where their route started and stopped and explain how these locations can be easily identified. (3 marks)

Solution

Start at F and stop at D (or reverse).
The graph is semi-Eulerian, which means that the graph contains an Euler trail but not an Euler circuit - hence start at one odd vertex and stop at the other.

Specific behaviours

✓ correct endpoints
✓ states graph is semi-Eulerian / has Euler trail
✓ indicates use of odd vertices

(c) Another inspector wishes to start in an area, follow a route that visits all the other areas exactly once and end up back where they started. Comment on whether this is possible, referring to the Hamiltonian properties of the graph in (a) to justify your response. (3 marks)

Solution

Not possible.
The graph is semi-Hamiltonian, which means that the graph contains a Hamilton path but not a Hamilton circuit - hence can visit all areas (vertices) just once but unable to return to start.

Specific behaviours

✓ states not possible
✓ states graph is semi-Hamiltonian / has Hamilton path
✓ explains meaning of semi-Hamiltonian

See next page

Question 19 **(7 marks)**

The average mid-year commuting times for full-time workers in Perth (p minutes) and Sydney (s minutes) between the years 2004 ($t = 4$) and 2011 ($t = 11$) are shown in the table below.

Year, t	4	5	6	7	8	9	10	11
Perth, p	24.2	25.8	27.2	26.1	27.7	27.1	30.2	29.5
Sydney, s	34.2	34.7	35.1	35.8	35.6	36.2	36.7	36.5

The least-squares line to model the linear relationship between t and s is $s = 0.348t + 33.0$ and $r_{ts} = 0.966$.

(a) Determine the least-squares line to model the linear relationship between t and p and state the correlation coefficient for this association. (2 marks)

Solution
$p = 0.719t + 21.8$ $r_{tp} = 0.902$
Specific behaviours
✓ correct equation, using correct variables ✓ correct value of r

(b) Predict the average commuting times in Perth and Sydney in the year 2021 and state, with justification, which prediction you are most confident in. (3 marks)

Solution
$p(21) = 0.719(21) + 21.8 \approx 36.9$ m $s(21) = 0.348(21) + 33.0 \approx 40.3$ m Most confident in prediction for Sydney as correlation is stronger. OR Not confident in either, as both involve considerable extrapolation.
Specific behaviours
✓ correct Perth time ✓ correct Sydney time ✓ justifies choice

(c) Predict the year in which the average commuting time will be the same in both cities and comment on how confident you are of this prediction. (2 marks)

Solution
$0.348t + 33.0 = 0.719t + 21.8$ $t \approx 30$ The times will be the same in year 2030. Not at all confident in this prediction as it involves considerable extrapolation.
Specific behaviours
✓ correct year ✓ no confidence justified using extrapolation

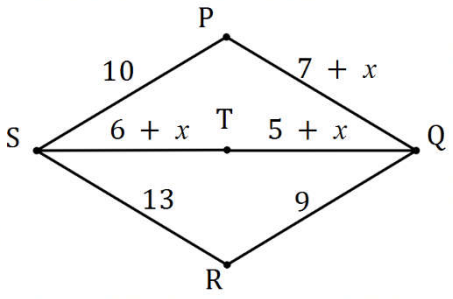
See next page

Question 20 **(7 marks)**

The edges in the graph (not to scale) represent roads and the weight on each edge is the time, in minutes, that it takes to drive along that road. The times to drive along PQ, ST and TQ vary throughout the day.

The variable x can only take whole number values.

An inspector wishes to drive along each road at least once, starting and finishing at P, in the minimum possible time.



(a) Briefly explain why the edges on a path between S and Q will have to be repeated. (1 mark)

Solution
S and Q are the only odd vertices.
Specific behaviours
✓ explanation

(b) List all possible paths between S and Q, and state how long each would take, in terms of x where appropriate. (2 marks)

Solution
SPQ - time is $17 + x$ STQ - time is $11 + 2x$ SRQ - time is 22
Specific behaviours
✓ at least two listed and correct ✓ all listed and correct

(c) Determine all possible values of x so that ST would be one of the repeated edges. (2 marks)

Solution
By substitution, STQ takes least time of three paths when: $x = 0, 1, 2, 3, 4, 5$.
Specific behaviours
✓ set of values with no more than 2 errors ✓ correct set of values

(d) For the case when $x = 7$, determine the time required for the inspectors' drive. (2 marks)

Solution
$SPQ + STQ + SRQ = 24 + 25 + 22 = 71$ Repeat SRQ, so an extra 22. Total time: $71 + 22 = 93$ minutes.
Specific behaviours
✓ sum of weights ✓ correct total time

See next page

Question 21 **(8 marks)**

A nail is hammered into a piece of wood. The distances moved by the tip of the nail during the first, second and third hits are 16, 12 and 9 mm respectively.

(a) Show that the distances can be modelled by a geometric sequence. (2 marks)

Solution
$r_1 = 12 \div 16 = 0.75$ $r_2 = 9 \div 12 = 0.75$ Hence the distances have a common ratio and can be modelled by a geometric sequence.
Specific behaviours
✓ both ratios correct ✓ states distances have a common ratio

(b) Write a rule for the distance moved by the tip of the nail D_n during the n^{th} hit of the hammer in the form $D_n = a(r)^{n-1}$. (1 mark)

Solution
$D_n = 16(0.75)^{n-1}$
Specific behaviours
✓ correct rule

(c) Determine which hit first moves the tip of the nail less than 3 mm, and state the distance moved during this hit, rounded to one decimal place. (2 marks)

Solution
$D_6 = 3.7969, \quad D_7 = 2.8477$ On the 7^{th} hit, when it moves 2.8 mm (1 dp).
Specific behaviours
✓ correct hit ✓ correct distance to 1 dp

(d) The piece of wood is 60 mm thick. State, with justification, whether the tip of the nail will pass all the way through the piece of timber, stating any assumptions that you make. (3 marks)

Solution
Yes. The nail will emerge on the 10^{th} hit as the sum of the first 10 terms is 60.4 mm. Assumptions: - geometric sequence will continue - nail is driven directly through wood, not at angle. - etc
Specific behaviours
✓ states yes (with justification) ✓ justifies with sum of terms ✓ at least one valid assumption

End of questions